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The stability of orbit of a planet in the field of binary stars

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In the paper, we consider the motion of a planet in the field of the binary stars in the theory of general relativity (GR), when all bodies have their own rotation. We consider that the third body moves in the plan of massive two bodies. The binary system orbit around the center of mass (sometimes referred to as the barycenter), in a circular orbit. The third body orbits closer to the primary star and does not affect the orbit of the secondary body. In fact, this problem belongs to the gravitational restricted three-body problem. We investigate the stability of the circular orbit of the planet in the binary system using the adiabatic theory of motion. The adiabatic theory of motion of bodies is the approach to study the evolutionary motion of bodies in the GR. The corresponding theory based on the vector elements of orbit to describe the motion of bodies and based on the asymptotic methods of nonlinear oscillations and in the method of adiabatic invariants.

Key words: General relativity, neutron stars, rotational motion, translational motion, stability of motion, three-body problem.
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1 Introduction

Binary and multiple stellar systems have importance in areas of astronomy and astrophysics. The component stars orbit each other, bound by gravitational attraction. The motion of a planet in the binary system can be considered as a restricted three-body problem, which has been a long history in classical mechanics[1-4]. The large of known binary star or triple stars systems with planets may actually be larger as some single stars, known to host planets, exhibit a drift in the γ-velocity, which indicates the presence of a distant companion. In classical theory, the planetary orbits in binary systems can be separated into three different categories as well as S-type, P-type, and L-type[5-8].

Currently, the motion of a planet in the binary system is mainly considered in the framework of General theory of relativity (GR). The characteristics of the orbit of the planet to be an important source of information about stellar internal structure as well as the possibility of verifying of General Relativity outside the Solar System. A strict and correct formulation of the motion of bodies in GR is more difficult than in classical mechanics, this is due to the complexity of the mathematical apparatus of GR.

The problem of the motion of bodies in GR can be split into two correlated problems, namely, the problem of the motion of point-like masses and the problem of the motion of extended bodies. There are several methods for obtaining the equations of motion from the equations of the gravitational field, such as the method of Einstein-Infeld-Hoffmann (EIH)[9], the first approximate method of Fock, the second approximate method of Fock[9,10], etc.

The motion of an extended body in GR mechanics first formulated by the V. Fok and after that developed by Petrova, Brumberg and Abdilldin[9-12]. Fock's methods make it possible to derive the equations of motion of extended bodies, allow us to take into account the internal structure and shape of bodies. In[12-21], this method used to investigate the problem of the orbital stability of a circular motion of a test body in the restricted three body problem in the GR, without solving the equations of motion.

In this paper, we consider evolutionary motion of a planet in the binary system within the framework of GR, which all bodes have own rotation.

2 The Lagrangian and Hamiltonian of the system

Consider that the motion of the binary system is circular and the orbit of the massless body lies in the same plane. We assume that the third mass is a planet and moving around the primary mass of the binary system and their entire have own rotations. The relativistic correction is compatible with the gravitational potentials of companions

$$U_1, U_2 << c^2, U_3 << U_1 \quad (1)$$

where – potentials of the primary and secondary bodies, respectively.

Assume that the inner structure of binary stars do not affect the evolutionary motion of planet and the planet motion do not affect to the motion of binary system. Using the first approximate method of Fock the Lagrange function of translational and rotational motion for the system can be presented as:

$$L = L^{(0)} + L^{(*)} \quad (2)$$

The Hamiltonian is

$$H = H^{(0)} + H^{(*)} \quad (3)$$

where first term is the Lagrange function and Hamiltonian for three point like masses, and the second term responsible for corrections containing rotational terms.

$$L^{(0)} = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} + \frac{m_3 v_3^2}{2} + \gamma \frac{m_1 m_2}{\left|\vec{r}_2 - \vec{r}_1\right|} + \gamma \frac{m_1 m_3}{\left|\vec{r}_3 - \vec{r}_1\right|} + \gamma \frac{m_2 m_3}{\left|\vec{r}_2 - \vec{r}_3\right|} + \frac{1}{8c^2}\left(m_1 v_1^4 + m_2 v_2^4 + m_3 v_3^4\right)$$
$$+ \frac{\gamma}{2c^2}\left[\frac{3m_1 m_2}{\left|\vec{r}_2 - \vec{r}_1\right|}\left(v_1^2 + v_2^2\right) + \frac{3m_1 m_3}{\left|\vec{r}_3 - \vec{r}_1\right|}\left(v_1^2 + v_3^2\right) + \frac{3m_2 m_3}{\left|\vec{r}_2 - \vec{r}_3\right|}\left(v_2^2 + v_3^2\right)\right]$$
$$+ \frac{\gamma}{c^2}\left[\frac{m_1 m_2}{\left|\vec{r}_2 - \vec{r}_1\right|}\left[7\left(\vec{v}_1 \vec{v}_2\right) + \frac{1}{\left(\left|\vec{r}_2 - \vec{r}_1\right|\right)^2}\left(\vec{v}_1\left(\vec{r}_2 - \vec{r}_1\right)\right)\left(\vec{v}_2\left(\vec{r}_2 - \vec{r}_1\right)\right)\right] + \right.$$
$$+ \frac{m_1 m_3}{\left|\vec{r}_3 - \vec{r}_1\right|}\left[7\left(\vec{v}_1 \vec{v}_3\right) + \frac{1}{\left(\left|\vec{r}_3 - \vec{r}_1\right|\right)^2}\left(\vec{v}_1\left(\vec{r}_3 - \vec{r}_1\right)\right)\left(\vec{v}_3\left(\vec{r}_3 - \vec{r}_1\right)\right)\right] +$$
$$\left. + \frac{m_2 m_3}{\left|\vec{r}_3 - \vec{r}_2\right|}\left[7\left(\vec{v}_2 \vec{v}_3\right) + \left(\vec{v}_2 \vec{n}_{23}\right)\left(\vec{v}_3 \vec{n}_{23}\right)\right]\right] - - \frac{\gamma^2 m_1 m_2 m_3}{c^2}\left(\frac{1}{\left|\vec{r}_2 - \vec{r}_1\right|\left|\vec{r}_3 - \vec{r}_1\right|} + \frac{1}{\left|\vec{r}_2 - \vec{r}_1\right|\left|\vec{r}_3 - \vec{r}_2\right|} + \frac{1}{\left|\vec{r}_1 - \vec{r}_3\right|\left|\vec{r}_2 - \vec{r}_3\right|}\right)$$
$$- \frac{\gamma^2}{2c^2}\left(\frac{m_1 m_2\left(m_1 + m_2\right)}{\left|\vec{r}_2 - \vec{r}_1\right|^2} + \frac{m_1 m_3\left(m_1 + m_3\right)}{\left|\vec{r}_3 - \vec{r}_1\right|^2} + \frac{m_3 m_2\left(m_3 + m_2\right)}{\left|\vec{r}_2 - \vec{r}_3\right|^2}\right), \quad (4)$$

The rotational term of Lagrangian is:

$$L^{*} = \sum_{i=1}^{3} \omega_i^2 I_i + \frac{1}{c^2}\left[\sum_{i=1}^{3} \omega_i^2 I_i - \frac{1}{2}\left(I_2\left(\vec{\omega}_2 \cdot \dot{\vec{r}}_2\right)^2 + I_3\left(\vec{\omega}_3 \cdot \dot{\vec{r}}_3\right)^2\right)\right]$$
$$- \frac{\gamma}{c^2} \cdot \left\{\frac{1}{\left|\vec{r}_2\right|^3}\left(\vec{r}_2 \cdot \dot{\vec{r}}_2\right) \cdot \left[3m_1 I_2 . \omega_2 + 4m_2 I_1 . \omega_1\right] - \frac{1}{\left|\vec{r}_3\right|^3}\left(\vec{r}_3 \cdot \dot{\vec{r}}_3\right) \cdot \left[3m_1 I_3 . \omega_3 + 4m_3 I_1 . \omega_1\right] + \right.$$

$$+\frac{\vec{r}_2-\vec{r}_3}{\left|\vec{r}_2-\vec{r}_3\right|^3}\cdot\left[\left(3\dot{\vec{r}}_3-4\dot{\vec{r}}_2\right)\cdot m_2 I_3.\omega_3-\left(3\dot{\vec{r}}_2-4\dot{\vec{r}}_3\right)\cdot m_3 I_2.\omega_2\right]\Bigg\}+\frac{\gamma}{2c^2}\cdot\frac{m_2 I_2+m_3 I_3}{\left|\vec{r}_2-\vec{r}_3\right|^3}\cdot$$

$$\cdot\left[-\dot{\vec{r}}_2\cdot\dot{\vec{r}}_3+3\frac{\left(\left(\vec{r}_2-\vec{r}_3\right)\cdot\dot{\vec{r}}_2\right)\cdot\left(\left(\vec{r}_2-\vec{r}_3\right)\cdot\dot{\vec{r}}_3\right)}{\left|\vec{r}_2-\vec{r}_3\right|^2}\right]-\frac{3\gamma}{c^2}\left[\frac{1}{\left|\vec{r}_2\right|}\cdot\left(m_1\omega_2^2 I_2+m_2\omega_1^2 I_1\right)+\right.$$

$$\left.+\frac{1}{\left|\vec{r}_3\right|}\left(m_1\omega_3^2 I_3+m_3\omega_1^2 I_1\right)+\frac{1}{\left|\vec{r}_2-\vec{r}_3\right|}\left(m_2\omega_3^2 I_3+m_3\omega_2^2 I_2\right)\right]+$$

$$+\frac{\gamma^2}{2c^2}\left\{\frac{m_1 I_2+m_2 I_1}{\left|\vec{r}_2\right|^3}+\frac{m_1 I_3+m_3 I_1}{\left|\vec{r}_3\right|^3}+\frac{m_2 I_3+m_3 I_2}{\left|\vec{r}_2-\vec{r}_3\right|^3}+\right.$$

$$\left.+m_1 m_2 I_3\cdot\frac{\vec{r}_3\cdot\left(\vec{r}_3-\vec{r}_2\right)}{\left|\vec{r}_3\right|^3\left|\vec{r}_2-\vec{r}_3\right|^3}+m_2 m_3 I_1\cdot\frac{\vec{r}_2\cdot\vec{r}_3}{\left|\vec{r}_2\right|^3\cdot\left|\vec{r}_3\right|^3}+m_1 m_3 I_2\cdot\frac{\left(\vec{r}_2-\vec{r}_3\right)\cdot\vec{r}_2}{\left|\vec{r}_2-\vec{r}_3\right|^3\left|\vec{r}_2\right|^3}\right\}-$$

$$-\frac{\gamma}{c^2}\left\{4\left[\frac{I_1 I_2}{\left|\vec{r}_2\right|^3}+\frac{I_1 I_3}{\left|\vec{r}_3\right|^3}+\frac{I_2 I_3}{\left|\vec{r}_2-\vec{r}_3\right|^3}\right]-3\left[\frac{\left(\vec{r}_2\cdot\vec{\omega}_1\right)\cdot\left(\vec{r}_2\cdot\vec{\omega}_2\right)}{\left|\vec{r}_2\right|^3}+\right.\right.$$

$$\left.\left.+\frac{\left(\vec{r}_3\cdot\vec{\omega}_1\right)\cdot\left(\vec{r}_3\cdot\vec{\omega}_3\right)}{\left|\vec{r}_3\right|^3}+\frac{\left(\left(\vec{r}_2-\vec{r}_3\right)\cdot\vec{\omega}_2\right)\cdot\left(\left(\vec{r}_2-\vec{r}_3\right)\cdot\vec{\omega}_3\right)}{\left|\vec{r}_2-\vec{r}_3\right|^3}\right]\right\}. \qquad (5)$$

The Hamiltonian H is determined by the equation:

$$H=\vec{v}_i\frac{\partial L}{\partial\vec{v}_i}-L$$

here $\vec{v}_i$ is the velocity of each companion of system.

3 Equation of motion

Our study based on asymptotic methods of nonlinear mechanics. We can write the equations of motion of the system in the representation of vector elements $\vec{M}$ (angular momentum) and $\vec{A}$ (Laplace vector):

$$\vec{M}=\left[\vec{r},\vec{p}\right] \qquad (6)$$

$$\vec{A}=\left[\frac{\vec{p}}{m},\vec{M}\right]-\gamma\frac{mm_0}{r}\vec{r} \qquad (7)$$

where

$$A=\gamma mm_0 e=\alpha e,$$

e – eccentricity of orbit.

The time derivative of angular momentum is

$$\dot{M}_i=\left[\dot{\vec{r}}_i,\vec{p}_i\right]+\left[\vec{r}_i,\dot{\vec{p}}_i\right] \qquad (8)$$

where

$$\dot{\vec{r}}_i=\frac{\partial H}{\partial p_i}=\frac{\partial H_0}{\partial p_i}+\frac{\partial H^*}{\partial p_i}=\dot{\vec{r}}_i^{(0)}+\dot{\vec{r}}_i^{(*)};\ \dot{\vec{p}}_i=-\frac{\partial H}{\partial r_i}=-\frac{\partial H_0}{\partial r_i}-\frac{\partial H^*}{\partial r_i}=\dot{\vec{p}}_i^{(0)}+\dot{\vec{p}}_i^{(*)}; \qquad (9)$$

$$\dot{\vec{r}}_3^{(0)}=\frac{\vec{p}_3}{m_3}\left[1-\frac{1}{c^2}\left(\frac{p_3^2}{m_3^2}+3\gamma\left(\frac{m_2}{\left|\vec{r}_2-\vec{r}_3\right|}+\frac{m_1}{\left|\vec{r}_3\right|}\right)\right)\right]+\frac{\gamma}{c^2}\frac{1}{\left|\vec{r}_2-\vec{r}_3\right|}\left(7\vec{p}_2+\frac{\left(\vec{r}_2-\vec{r}_3\right)\left(\vec{p}_2\left(\vec{r}_2-\vec{r}_3\right)\right)}{\left|\vec{r}_2-\vec{r}_3\right|^2}\right) \qquad (10)$$

After calculations, the equation of motion is

$$\dot{\vec{p}}_3^{(0)} = -\gamma\frac{m_1m_3}{|\vec{r}_3|^3}\vec{r}_3 + \gamma\frac{m_2m_3}{|\vec{r}_2-\vec{r}_3|^3}(\vec{r}_2-\vec{r}_3) - \frac{7\gamma}{2c^2}\frac{(\vec{p}_2\vec{p}_3)(\vec{r}_2-\vec{r}_3)}{|\vec{r}_2-\vec{r}_3|^3}$$
$$-\frac{\gamma}{2c^2}\left[\frac{3(\vec{p}_3(\vec{r}_2-\vec{r}_3))(\vec{p}_2(\vec{r}_2-\vec{r}_3))(\vec{r}_2-\vec{r}_3)}{|\vec{r}_2-\vec{r}_3|^5} - \frac{1}{|\vec{r}_2-\vec{r}_3|^3}\left[\vec{p}_2(\vec{p}_3(\vec{r}_2-\vec{r}_3)) + \vec{p}_3(\vec{p}_2(\vec{r}_2-\vec{r}_3))\right]\right]$$
$$+\frac{3\gamma}{2c^2}\left[\frac{\vec{p}_2^{\,2}}{m_2}\frac{m_3(\vec{r}_2-\vec{r}_3)}{|\vec{r}_2-\vec{r}_3|^3} + \frac{\vec{p}^{\,2}}{m_3}\left(\frac{m_2(\vec{r}_2-\vec{r}_3)}{|\vec{r}_2-\vec{r}_3|^3} - \frac{m_1\vec{r}_3}{|\vec{r}_3|^3}\right)\right]$$
$$-\frac{\gamma^2m_1m_2m_3}{c^2}\left[-\frac{\vec{r}_3}{|\vec{r}_2||\vec{r}_3|^3} - \frac{\vec{r}_2-\vec{r}_3}{|\vec{r}_2-\vec{r}_3|^3|\vec{r}_2|} - \frac{\vec{r}_3}{|\vec{r}_2-\vec{r}_3||\vec{r}_3|^3} + \frac{\vec{r}_2-\vec{r}_3}{|\vec{r}_2-\vec{r}_3|^3|\vec{r}_3|}\right]$$
$$+\frac{\gamma^2}{c^2}\left[\frac{m_1m_3(m_1+m_3)\vec{r}_3}{|\vec{r}_3|^4} - \frac{m_2m_3(m_2+m_3)(\vec{r}_2-\vec{r}_3)}{|\vec{r}_2-\vec{r}_3|^4}\right] \tag{11}$$

$$\dot{\vec{r}}_3^{(*)} = \frac{1}{c^2}I_3(\vec{\omega}_3\cdot\vec{p}_3)\vec{\omega}_3 - \frac{\gamma}{2c^2}\cdot\frac{m_2I_2+m_3I_3}{|\vec{r}_2-\vec{r}_3|^3}\cdot\left[-\vec{p}_2 + 3\frac{((\vec{r}_2-\vec{r}_3)\cdot\vec{p}_2)\cdot(\vec{r}_2-\vec{r}_3)}{|\vec{r}_2-\vec{r}_3|^2}\right]. \tag{12}$$

$$\dot{\vec{p}}_3^{(*)} = \left\{-\frac{3\gamma}{2c^2}(m_2I_2+m_3I_3)\frac{\vec{p}_2\vec{p}_3}{|\vec{r}_2-\vec{r}_3|^5} + \frac{15\gamma}{2c^2}\frac{((\vec{r}_2-\vec{r}_3)\vec{p}_2)\cdot((\vec{r}_2-\vec{r}_3)\vec{p}_3)}{|\vec{r}_2-\vec{r}_3|^7} -\right.$$
$$-\frac{3\gamma}{c^2}\frac{1}{|\vec{r}_2-\vec{r}_3|^3}(m_2\vec{\omega}_3^2I_3 + m_3\vec{\omega}_2^2I_2) + \frac{3\gamma^2}{2c^2}\frac{(m_2I_3+m_3I_2)}{|\vec{r}_2-\vec{r}_3|^5} +$$
$$+\frac{\gamma^2}{2c^2}m_1m_2I_3\frac{1}{|\vec{r}_3|^3|\vec{r}_2-\vec{r}_3|^3} - \frac{3\gamma^2}{2c^2}m_1m_2I_3\frac{(\vec{r}_3(\vec{r}_2-\vec{r}_3))}{|\vec{r}_3|^3|\vec{r}_2-\vec{r}_3|^3} - \frac{\gamma^2}{2c^2}m_2m_3I_1\frac{1}{|\vec{r}_2|^3|\vec{r}_3|^3} +$$
$$\left.-\frac{9((\vec{r}_2-\vec{r}_3)\vec{\omega}_2)((\vec{r}_2-\vec{r}_3)\vec{\omega}_3)}{|\vec{r}_2-\vec{r}_3|^5}\right\}\vec{r}_2 + \left[\vec{r}_3,\left\{-\frac{9\gamma}{2c^2}(m_2I_2+m_3I_3)\frac{\vec{p}_2\vec{p}_3}{|\vec{r}_2-\vec{r}_3|^5} -\right.\right.$$
$$-\frac{15\gamma}{2c^2}\frac{((\vec{r}_2-\vec{r}_3)\vec{p}_2)\cdot((\vec{r}_2-\vec{r}_3)\vec{p}_3)}{|\vec{r}_2-\vec{r}_3|^7} + \frac{3\gamma}{c^2}\frac{1}{|\vec{r}_3|^3}(m_1\vec{\omega}_3^2I_3 + m_3\vec{\omega}_1^2I_1) +$$
$$+\frac{3\gamma}{c^2}\frac{1}{|\vec{r}_2-\vec{r}_3|^3}(m_2\vec{\omega}_3^2I_3 + m_3\vec{\omega}_2^2I_2) - \frac{3\gamma^2}{2c^2}\frac{(m_1I_3+m_3I_1)}{|\vec{r}_3|^5} - \frac{3\gamma^2}{2c^2}\frac{(m_2I_3+m_3I_2)}{|\vec{r}_2-\vec{r}_3|^5} -$$
$$-\frac{\gamma^2}{c^2}m_1m_2I_3\frac{1}{|\vec{r}_3|^3|\vec{r}_2-\vec{r}_3|^3} - \frac{3\gamma^2}{2c^2}m_1m_2I_3\frac{(\vec{r}_3(\vec{r}_2-\vec{r}_3))}{|\vec{r}_3|^5|\vec{r}_2-\vec{r}_3|^3} - \frac{3\gamma^2}{2c^2}m_1m_2I_3\frac{(\vec{r}_3(\vec{r}_2-\vec{r}_3))}{|\vec{r}_3|^3|\vec{r}_2-\vec{r}_3|^3} +$$
$$+\frac{3\gamma^2}{2c^2}m_2m_3I_1\frac{(\vec{r}_2\vec{r}_3)}{|\vec{r}_2|^3|\vec{r}_3|^5} + \frac{3\gamma^2}{2c^2}m_1m_3I_2\frac{((\vec{r}_2-\vec{r}_3)\vec{r}_2)}{|\vec{r}_2-\vec{r}_3|^5|\vec{r}_2|^3} + \frac{12\gamma}{c^2}\frac{I_1I_3}{|\vec{r}_3|^5} + \frac{12\gamma}{c^2}\frac{I_2I_3}{|\vec{r}_2-\vec{r}_3|^5} +$$

$$+\frac{\gamma}{c^2}\frac{9\left(\left(\vec{r}_2-\vec{r}_3\right)\vec{\omega}_2\right)\left(\left(\vec{r}_2-\vec{r}_3\right)\vec{\omega}_3\right)\vec{r}_3}{\left|\vec{r}_2-\vec{r}_3\right|^5}\Bigg\}\vec{r}_3\Bigg]+\Bigg[\vec{r}_3,\frac{\gamma}{c^2}\Bigg\{\frac{9\left(\vec{r}_3\vec{\omega}_2\right)\left(\vec{r}_3\vec{\omega}_3\right)}{\left|\vec{r}_2-\vec{r}_3\right|^3}\vec{r}_3-\frac{3\left(\vec{r}_3\vec{\omega}_3\right)}{\left|\vec{r}_3\right|^3}\vec{\omega}_1+\frac{3\left(\vec{r}_3\omega_1\right)}{\left|\vec{r}_3\right|^3}\vec{\omega}_3+$$
$$+\frac{3\left(\left(\vec{r}_2-\vec{r}_3\right)\vec{\omega}_3\right)}{\left|\vec{r}_2-\vec{r}_3\right|^3}\vec{\omega}_2+\frac{3\left(\left(\vec{r}_2-\vec{r}_3\right)\vec{\omega}_2\right)}{\left|\vec{r}_2-\vec{r}_3\right|^3}\vec{\omega}_3\Bigg\}\Bigg]. \quad (13)$$

where I_i – moment of inertia, $\vec{\omega}_i$ – orbital angular velocity. Bearing in mind (8), we obtained the derivatives the equation (8) can be represent in the following form:

$$\dot{M}=\dot{M}^{(0)}+\dot{M}^{(*)} \quad (14)$$

where $\dot{M}^{(0)}$ is the time derivative of the momentum for three masses, and the $\dot{M}^{(*)}$ is related to the rotational term.

4 Integration of equation of motions

In the reason to obtain the equation of evolutionary motion, it is necessary to integrate equation (8) over the period of repetition of the configurations of the system T (the synodic period of the test body):

$$\bar{\dot{M}}=\frac{1}{T}\int_0^T\left(\bar{\dot{M}}^{(0)}+\bar{\dot{M}}^{(*)}\right)dt \quad (15)$$

where

$$T=\frac{2\pi}{\omega_2-\omega_3}. \quad (16)$$

In general, to integrate equation (15) is complicated task. Let's consider the case when the angular velocity $\vec{\omega}_i$ $(i=2,3)$ a rotating body is perpendicular to its orbital plane: i.e.,:

$$\vec{\omega}_i=\vec{k}\vec{\omega}_z^{(i)}(i=1,2,3), \quad (17)$$

$$\vec{r}_i=\vec{i}x^{(i)}+\vec{j}y^{(i)},\ (i=1,2,3) \quad (18)$$

and

$$\vec{p}_i=\vec{i}p^{(i)}{}_x+\vec{j}p^{(i)}{}_y,\ (i=1,2,3), \quad (19)$$

Then, taking into account (17), (18) and (19), the equation for the desired vector function takes the form:

$$\dot{M}^{(0)}=\Bigg\{\gamma m_2m_3-\frac{7\gamma}{2c^2}\left(\vec{p}_2\vec{p}_3\right)-\frac{3\gamma}{2c^2}\frac{\left(\vec{p}_2\left(\vec{r}_2-\vec{r}_3\right)\right)\left(\vec{p}_3\left(\vec{r}_2-\vec{r}_3\right)\right)}{\left|\vec{r}_2-\vec{r}_3\right|^2}$$
$$+\frac{3\gamma m_2m_3}{2c^2}\left(\frac{\vec{p}_2^{\,2}}{m_2^2}+\frac{\vec{p}_3^{\,2}}{m_3^2}\right)-\frac{\gamma^2}{c^2}m_1m_2m_3\left(\frac{1}{\left|\vec{r}_2\right|}+\frac{1}{\left|\vec{r}_3\right|}\right)\Bigg\}\frac{\left[\vec{r}_3,\vec{r}_2\right]}{\left|\vec{r}_2-\vec{r}_3\right|^3}$$
$$+\frac{\gamma}{2c^2}\left(\vec{p}_3\left(\vec{r}_2-\vec{r}_3\right)\right)\frac{\left[\vec{r}_3,\vec{p}_2\right]}{\left|\vec{r}_2-\vec{r}_3\right|^3}-\frac{\gamma}{2c^2}\left(\vec{p}_2\left(\vec{r}_2-\vec{r}_3\right)\right)\frac{\left[\vec{r}_2,\vec{p}_3\right]}{\left|\vec{r}_2-\vec{r}_3\right|^3}$$
$$-\frac{\gamma}{c^2}\frac{\left(\vec{p}_2\left(\vec{r}_2-\vec{r}_3\right)\right)}{\left|\vec{r}_2-\vec{r}_3\right|^3}\vec{M}+\frac{7\gamma}{2c^2}\frac{\left[\vec{p}_2,\vec{p}_3\right]}{\left|\vec{r}_2-\vec{r}_3\right|} \quad (20)$$

$$\dot{M}^{(*)}=-\frac{\gamma}{2c^2}\cdot\frac{m_2I_2+m_3I_3}{\left|\vec{r}_2-\vec{r}_3\right|^3}\cdot\left\{\left[-\vec{p}_2,\vec{p}_3\right]+3\frac{\left(\left(\vec{r}_2-\vec{r}_3\right)\cdot\vec{p}_2\right)}{\left|\vec{r}_2-\vec{r}_3\right|^2}\left[\left(\vec{r}_2-\vec{r}_3\right),\vec{p}_3\right]\right\}+$$
$$+\Bigg\{-\frac{3\gamma}{2c^2}\left(m_2I_2+m_3I_3\right)\frac{\vec{p}_2\vec{p}_3}{\left|\vec{r}_2-\vec{r}_3\right|^5}+\frac{15\gamma}{2c^2}\frac{\left(\left(\vec{r}_2-\vec{r}_3\right)\vec{p}_2\right)\cdot\left(\left(\vec{r}_2-\vec{r}_3\right)\vec{p}_3\right)}{\left|\vec{r}_2-\vec{r}_3\right|^7}-$$

$$-\frac{3\gamma}{c^2}\frac{1}{\left|\vec{r}_2-\vec{r}_3\right|^3}\left(m_2\vec{\omega}_3^2 I_3+m_3\vec{\omega}_2^2 I_2\right)+\frac{3\gamma^2}{2c^2}\frac{\left(m_2 I_3+m_3 I_2\right)}{\left|\vec{r}_2-\vec{r}_3\right|^5}+$$
$$+\frac{\gamma^2}{2c^2}m_1 m_2 I_3\frac{1}{\left|\vec{r}_3\right|^3\left|\vec{r}_2-\vec{r}_3\right|^3}-\frac{3\gamma^2}{2c^2}m_1 m_2 I_3\frac{\left(\vec{r}_3\left(\vec{r}_2-\vec{r}_3\right)\right)}{\left|\vec{r}_3\right|^3\left|\vec{r}_2-\vec{r}_3\right|^3}-\frac{\gamma^2}{2c^2}m_2 m_3 I_1\frac{1}{\left|\vec{r}_2\right|^3\left|\vec{r}_3\right|^3}+$$
$$\left.-\frac{\gamma}{c^2}\frac{9\left(\left(\vec{r}_2-\vec{r}_3\right)\vec{\omega}_2\right)\left(\left(\vec{r}_2-\vec{r}_3\right)\omega_3\right)}{\left|\vec{r}_2-\vec{r}_3\right|^5}\right\}\left[\vec{r}_3,\vec{r}_2\right]. \quad (21)$$

In our case, the binary stars have a circular orbit, we can write the radius vector of the bodes as a following

$$\vec{r}_\alpha = r_{kep}\left(\vec{i}\cos\omega_\alpha t+\vec{j}\sin\omega_\alpha t\right), \quad (\alpha=1,2,3) \quad (22)$$

and also the impulses as derivatives of them multiplied by the corresponding masses, after that we can integrate over the period T (16).

Calculating (21) and (22), the result shows:

$$\bar{\dot{\vec{M}}}=\frac{1}{T}\int_0^T \dot{M}dt=0. \quad (23)$$

The equation (23) is the sum of the vectors of the orbital moments of three bodies in the closed system is preserved. The conservation of the vector $\bar{\dot{\vec{M}}}$ follows that the circular motion of a rotating body in the field of massive binary stars, provided $\vec{\omega}_i=\vec{k}\omega^{(i)}{}_z\,(i=2,3)$ is stable.

5 Conclusions

In this work, we considered the orbital stability problem for the circular motion of a planet in the binary system within the framework of GR, with their own rotations.

The Lagrange function presented with the relativistic term and with the rotational terms then we investigated the stability problem of the orbit using by the adiabatic theory of motion in the GR.

The stability problem investigated by the time derivative of momentum vector, in the special case, if the spins of all bodies are collinear within a plane, then the circular motion of bodes will be stable. The orbital stability of quasi-circular orbits of the test body is a task of further investigations.

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